Determining Lengths and Optical Parameters for Dual Recycling at the 40m LIGO Prototype

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Abstract

The next phase of the LIGO project will allow tuning of a specific frequency to amplify the gravitational wave signal. The LIGO II design accomplishes this by using an extra mirror to implement a dual recycling configuration. Appropriate parameters (mirror reflectivities, cavity lengths, RF modulation frequencies) need to be determined to implement and test this setup at the 40m LIGO Prototype. Twiddle, a Mathematica program for modeling LIGO-like interferometers, was used to find lengths and optical parameters that optimize the frequency response of the interferometer and provide a functional length-sensing scheme.

Introduction

The Laser Interferometer Gravitational-wave Observatory (LIGO) is a facility dedicated to the detection of cosmic gravitational waves. The construction and operation of LIGO is a collaborative effort between Caltech and MIT with funding from the National Science Foundation (NSF). The two detectors that make up LIGO are located in Hanford, WA and Livingston, LA. A prototype LIGO detector, the 40m LIGO prototype, is located at Caltech and is used primarily to test optical and controls configurations for the LIGO detectors.

Gravitational waves are ripples in space-time predicted by Einstein's 1916 general theory of relativity. However, these waves have never been detected and their direct measurement is the main purpose of LIGO. A gravitational wave is emitted by accelerating masses, and a large gravity wave with a detectable signal could be emitted by interacting black holes or neutron stars. A gravitational wave interacts with LIGO by distorting space-time in a very specific way. The LIGO detectors are designed to measure this distortion as accurately as possible.

At the current stage of the LIGO project, the detectors are power-recycled Michelson interferometers with 4km long Fabry-Perot arms. This optical configuration, shown in Figure 1, measures a gravitational wave's distortion of space-time as a difference in the lengths of its two arms. It does this with the basic properties of a Michelson interferometer. If the two arms of a Michelson interferometer are the exact same length, light coming back from the arms will interfere constructively in one direction and destructively in the other direction, creating a bright port and a dark port. If the arms differ in length by even a small amount, this phenomenon ceases to occur and a signal can be detected at the dark port.

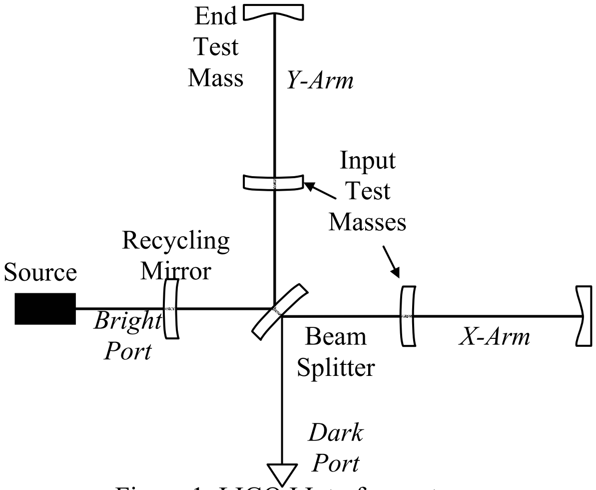


Figure 1: LIGO I Interferometer

The magnitude of this signal is greatly dependent upon two factors, the input light power and the magnitude of the length change in the arms. Placing a mirror at the bright port can increase the light power. This technique recycles light back into the interferometer and is called power-recycling. Amplifying the length change in the arms can be accomplished by simply making the arms longer. Rather than make the arms physically longer, the optical path can be folded on top of itself by placing an input mirror in each arm, creating Fabry-Perot arm cavities. The two mirrors that make up the Fabry-Perot arms are the called the test masses – they move in response to changes in the gravitational field and cause the length change that provides a gravitational wave signal. These are the basic components that define the optical configuration of the LIGO I detector.

Since very small changes in the lengths of the arms cause a signal to appear in the dark port, the interferometer is very sensitive to spurious mirror movements. Thus, there must be some scheme that can control this noise and isolate the gravitational wave signal. The entire optical setup sits inside a vacuum on a seismic isolation stack, so using any external components to monitor this noise would disturb this noise-free environment. Thus, the sensing system must utilize only the optics already in the interferometer. In 1991, S. Whitcomb proposed a scheme that could accomplish this task. By phase modulating the laser light at a radio frequency, two sidebands could be put on the light, one above the carrier frequency and one below it. The lengths of various parts of the interferometer are then adjusted to set the resonance characteristics of the sidebands and the carrier. Then, by monitoring the carrier and sideband signals at different parts of the interferometer, the distances between all the mirrors can be monitored.

To understand how this works, the interferometer needs to be broken down into three cavities, shown in Figure 2. Between the power-recycling mirror and the input test masses is the power-recycling cavity. The two Fabry-Perot arms are the other two cavities. Depending on the length of a cavity, a certain frequency light will have a specific tune. At the two extremes, light is resonant: it interferes constructively with itself and causes a power gain; or light is anti-resonant: it interferes destructively and causes a power loss. Consequently, a small length change near resonance will cause a large change in light power while any length change near anti-resonance will cause very little change in power. These facts allow the sidebands to provide information about the distances between mirrors in the interferometer.

Since there is no way to tell whether the lengthening or shortening of a single arm is due to a gravitational wave or some noise source, the distances between the mirrors needs to be broken down into degrees of freedom that separate out the gravitational wave signal. The effect that a gravitational wave will have on the LIGO interferometer is a lengthening of one arm coinciding with a shortening of the other arm. This degree of freedom is designated L-, the differential length of the arms. To define the lengths of the arms, another degree of freedom, L+, the common length of the arms, must be used. Any change in L+ must be due to noise as a gravitational wave cannot cause this sort of motion. Within the power-recycling cavity, there are corresponding l+ and l-, common and differential Michelson degrees of freedom, that define the positions of the other mirrors.

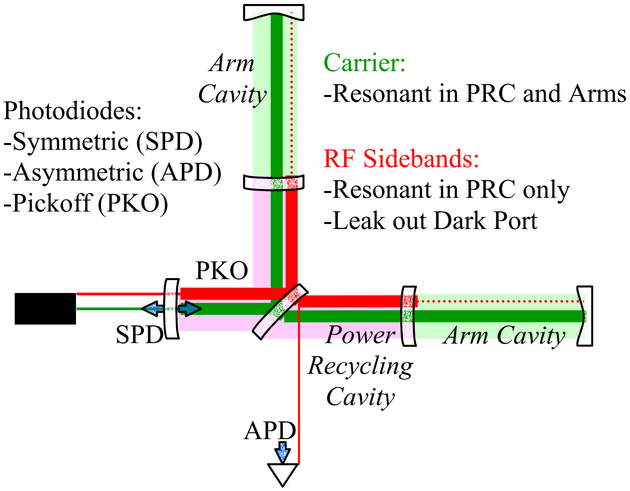


Figure 2: LIGO I Length Sensing

Differentiating between these signals using only the carrier and the sidebands is rather complicated, but relies on fixing the resonance conditions of the sidebands in the cavities. The carrier is resonant in both the power-recycling and arm cavities. The sidebands on the other hand, are resonant in the power-recycling cavity but anti-resonant in the arms. Also, some asymmetry is put into the interferometer so that the sidebands leak out the dark part. With these characteristics, the values of the electric field at a few points in the interferometer can reveal the values of each of the degrees of freedom.

In the LIGO I length sensing scheme, summarized in Figure 2, photodiodes are placed before the recycling mirror, inside the power-recycling cavity, and at the dark port – the symmetric, pickoff, and asymmetric ports, respectively. At each port, the signals from the sidebands and carrier can be demodulated at two phases, inphase and quadphase. The L+ and l+ signals come from the inphase symmetric and pickoff signals. The L- signal comes from the quadphase asymmetric, and the l- from the quadphase symmetric and pickoff signals.

In the next stage of the LIGO project, LIGO II, the detectors will have a different optical layout. The LIGO II detectors will be *dual*-recycled Michelson interferometers with Fabry-Perot arms. This will entail the addition of a signal-recycling mirror at the dark port of the LIGO I configuration. This mirror will increase the power of the gravitational wave signal at the dark port, making smaller L- signals detectable. This additional mirror adds another cavity, the signal-recycling cavity, and its length must be monitored along with the arms and the power-recycling cavity. A scheme for measuring these lengths requires additional sidebands, which makes the control scheme much more complicated.

The main benefit of the dual-recycling scheme is the ability to recycle a gravitational wave signal. However, it can only recycle one frequency of signal at a time, so it is most useful for events with known frequencies, such as pulsars. A frequency that is resonant in the signal-recycling cavity will be recycled. Thus, the length of the signal-recycling cavity can be varied to detune the interferometer to a certain frequency. This will multiply the signal by a significant amount and will allow many previously undetectable signals to be seen.

This LIGO II configuration must be implemented at the 40m prototype with a working length sensing and control system. This will verify the feasibility of this system for use at the LIGO 4km detectors. One length-sensing scheme, developed by Jim Mason, uses a set of RF sidebands and a frequency-shifted subcarrier. The sidebands are resonant in the power-recycling cavity and the subcarrier is resonant in both the power-recycling cavity and the signal. Using the same photodiodes, all the relevant degrees of freedom can be monitored.

The current plan uses the same three photodiodes as the LIGO I scheme but uses two pairs of RF sidebands, one with a modulation frequency several times the other one. One set of sidebands is resonant in the power-recycling cavity only and the other set of sidebands is resonant in both the power-recycling and signal-recycling cavities. They are both anti-resonant in the arms. This configuration allows the measurement of each of the relevant degrees of freedom.

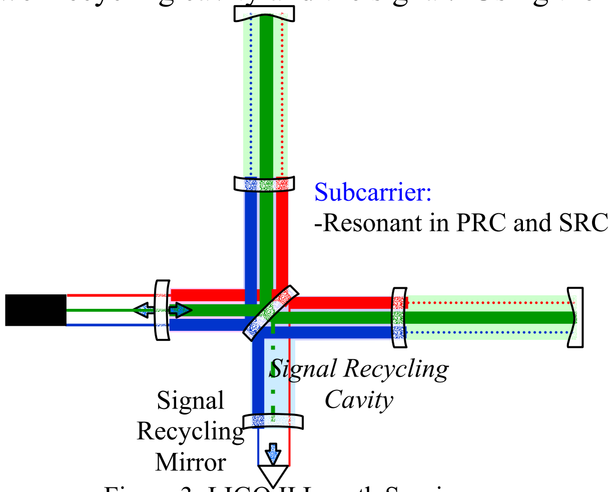


Figure 3: LIGO II Length Sensing

Materials and Methods

Before any optical configuration can be implemented at the 40m, a mathematical model must be created to predict the length sensing signals that will be expected. A Mathematica package called Twiddle, written by James Regehr, James Mason, and Hiro Yamamoto for modeling LIGO-like interferometers, can be used to simulate the response of a particular interferometer configuration to different mirror motions. Within the model, various optical parameters and lengths can be varied to optimize the response of the interferometer to the degrees of freedom which are important. Only after these signals are optimized is the optical configuration implemented in a real interferometer.

The parameters inputted into a Twiddle model are the modulation frequencies of the sidebands, the tune of the carrier light in each cavity, the layout, optical parameters, and distances between each mirror. These parameters define the static interferometer and Twiddle can calculate the electric fields at every position in the interferometer. Then, mirrors in the interferometer can be shaken at specific frequencies and electric fields can be monitored to find transfer functions. Cavities can also be swept to find the DC error signals for the different degrees of freedom.

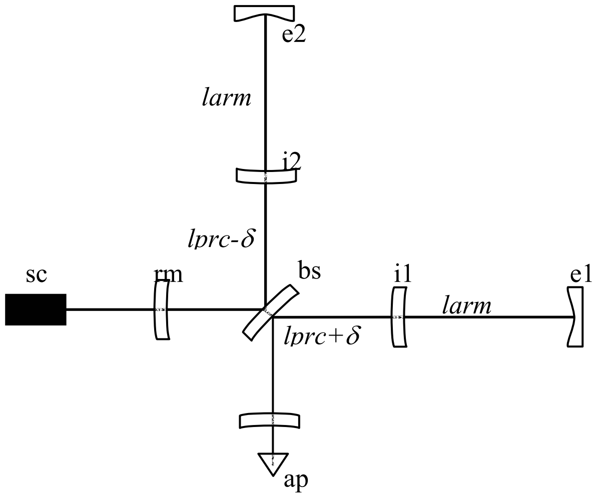
For the model of Jim Mason's length sensing scheme, the following parameters were entered into Twiddle: The n symbol represents the detuning of the interferometer, which ranges from 0 to 1.

Source	Freq (MHz)	Amplitude
SB-	-36.6868	0.235667i
Carrier	0	0.912898
SB+	36.6868	0.235667i
SubCarr	110.06	0.235667i

Mirror	Reflect	Trans	Loss
Recycl	*0.8*	0.2	2E-5
BmSpl	*0.5*	0.5	7.5E-4
ITM	*0.97*	0.03	2E-5
ETM	*1*	1.5E-5	2E-5
Signal	*0.8*	0.2	2E-5

Lengths	
lprc	2.04292m
δ	0.337081m
larm	38.8154m
lsrc	$c/4\times(110.06)\times(5-\nu)$

Tune ($\times\pi/2$)	PRC	Arms	SRC
Carrier	2n	2n+1	ν
SB – Carr	3	19	(5–ν)/3
Sub – Carr	9	57	5–ν

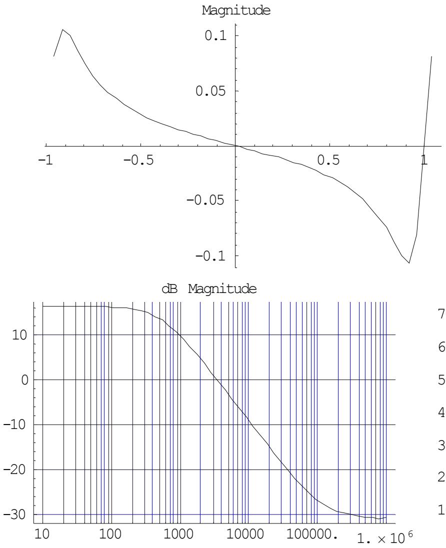


Results

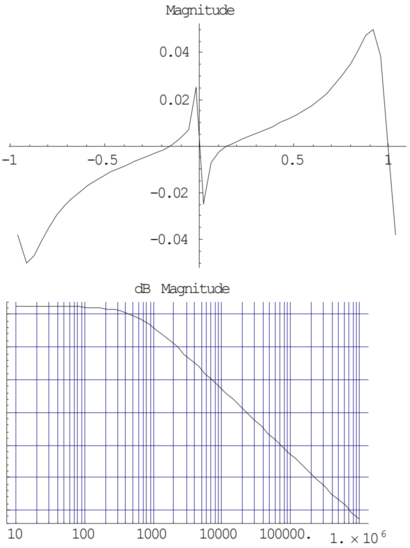
The Twiddle model can simulate longitudinal motion of any mirror and provide the output at any position in the interferometer. This allows the model to provide the signals that are associated with each degree of freedom. Two graphs can be generated, a DC error signal, resulting from a sweep of the cavity length; and a Bode plot, showing the slope of that error signal at different frequencies of mirror motion. All units of cavity length are in wavelengths of the carrier light and the powers are in $W/(\lambda/2\pi)$. Here are the Error signals and Bode plots for each degree of freedom:

Common Arm (L+):

SPD

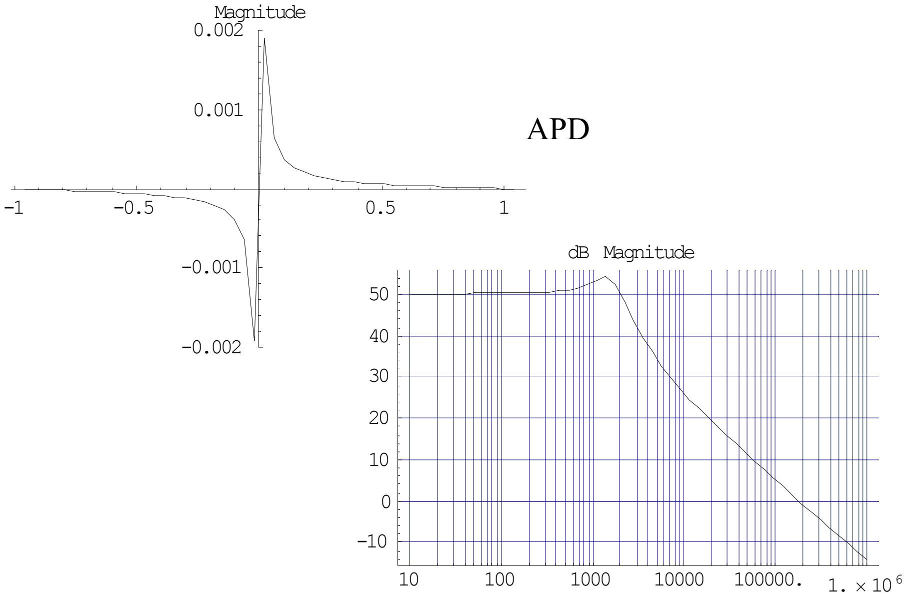


PKO



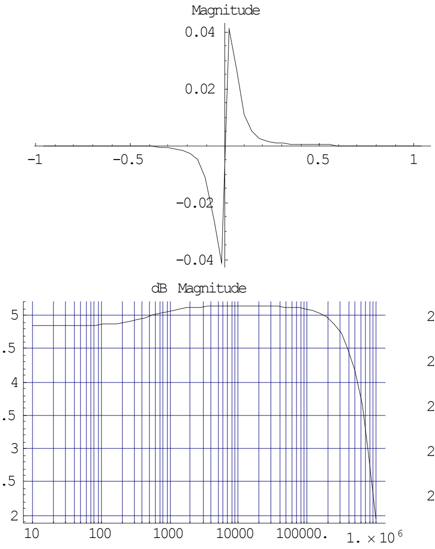
Differential Arm (L-):

APD



Common Michelson (l+):

SPD



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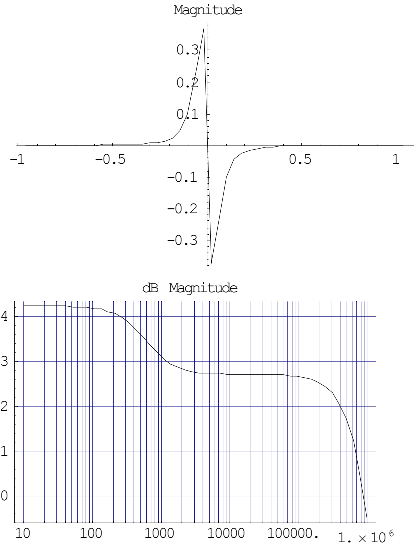
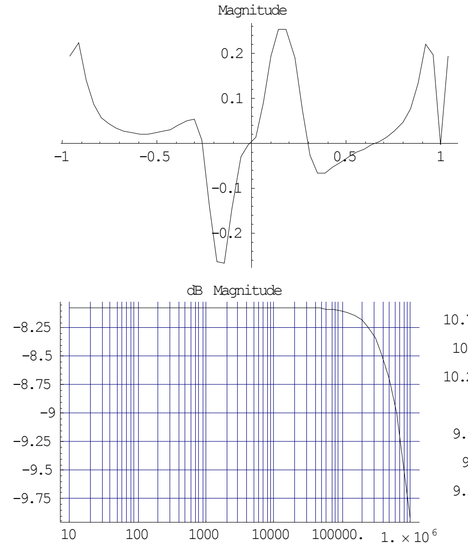
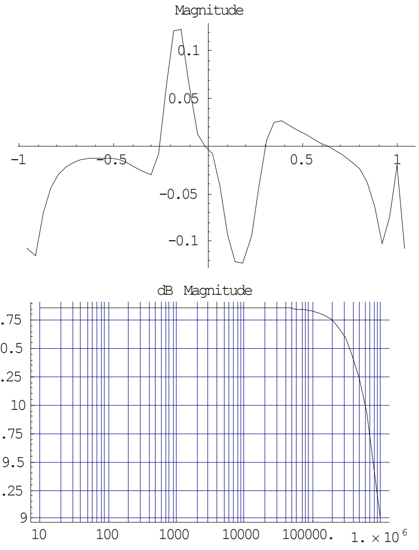


Differential Michelson (l-):

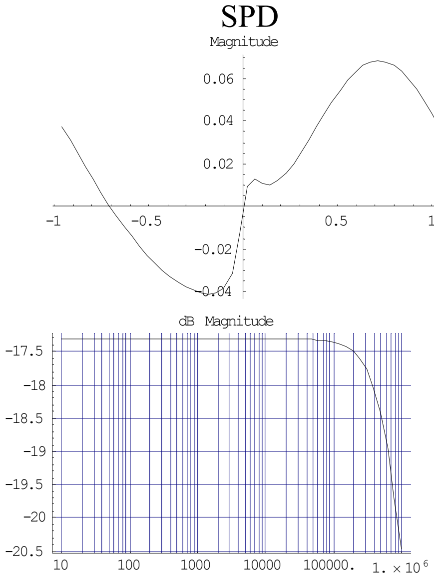
SPD

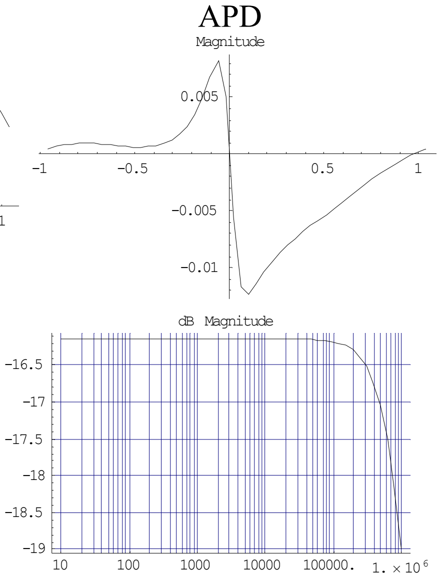


PKO



Common Signal Recycling (s+)





The slopes of the DC plots gives the magnitude of the error signal that can be expected. These values can be put into a matrix. A diagonal matrix will allow the signals to be distinguished most clearly. The matrix for this configuration is nearly diagonal:

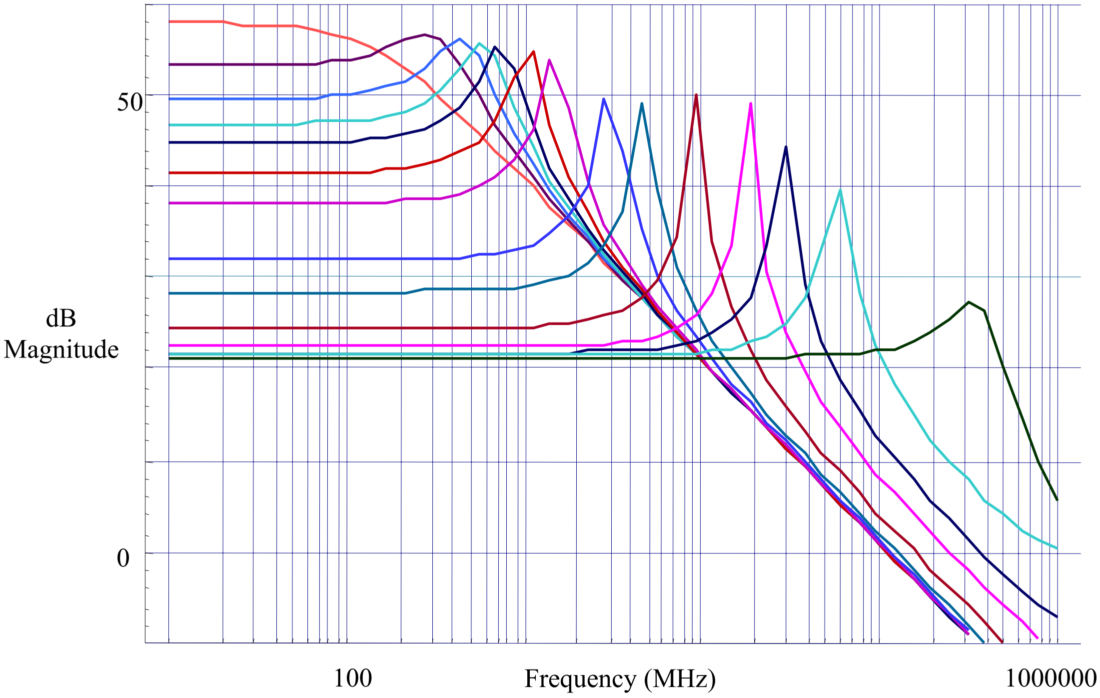
Freq	Phase	PD	L+	L-	l+	l-	s+
36.6868 MHz	0.19π	SPD	1516.95	0.014876	-9.33915	**1.95352**	-0.565124
		PKO	-922.376	0.13147	26.5742	**17.2651**	-4.99453
	0	APD	0	56.4169	0	0.429603	0
110.06 MHz		SPD	**-8.19106**	0	**-1.74752**	0	1.75378
		PKO	**-4230.34**	0	**16.2174**	0	15.4998
	0.39π	APD	0	**324.076**	0	2.46777	0
73.3736 MHz	1.78π	SPD	0.000189	0.000033	0.111518	0.004317	**0.136336**
	1.32π	PKO	0.026592	-0.00327	-2.18296	-0.429417	1.3092
	0.11π	APD	-0.00179	-0.00044	0.079241	-0.057963	**-0.155943**

The signal-recycling properties of this optical configuration can also be investigated with the Twiddle model. By varying the length of the signal-recycling cavity, the interferometer can be detuned to different gravitational wave frequencies. By looking at the Bode plot for the L- signal, this detuning can be seen as a peak in the resonance peak in the graph. Here is a chart of various SRC lengths and detunings:

ν	1	0.98	0.97	0.96	0.95	0.93	0.9
s+ (m)	2.7239	2.7375	2.7443	2.7511	2.7579	2.7716	2.7920
Pole (kHz)	0	0.28	0.43	0.53	0.67	1.1	1.4

ν	0.8	0.7	0.5	0.3	0.2	0.1	0
s+ (m)	2.8601	2.9282	3.0644	3.2006	3.2687	3.3368	3.4049
Pole (kHz)	2.8	3.4	9.5	18	30	60	300

And, a graph of their respective Bode plots:



Discussion

The parameters used in the Twiddle model were able to provide workable length sensing signals and exhibit signal-recycling behavior across a wide range of frequencies. This shows that this scheme is good enough to implement at the 40m interferometer.

The error signals all show an approximately linear behavior around perfect alignment, which makes it easier to design feedback systems to stabilize the lengths of the cavities when they fall away from their desired value. In most of the error signals, there is a minimum of other zero crossings, so that it will be apparent to the control plant that it is centering the length of the cavity on the correct value.

The DC Matrix shows 0's and small numbers for many signals that are not important, which is good because it reduces the amount of spurious signals that can enter into each specific error signal. The numbers in bold are the signals that appear most useful for each degree of freedom. In most cases, they are the largest values in their row, so that if a signal appears at a particular frequency and phase, it can be identified with the appropriate degree of freedom. In cases where to signals share a row, a gain hierarchy can be set up to filter out the smaller signal.

The signal-recycling characteristics show a well-defined peak for a very wide range of frequencies. At the 40m however, only the frequencies below around 1000 MHz will have visible peaks because other noise sources obscure the higher frequencies. Thus, the 40m dual-recycling configuration will have to operate very near the upper limit of detunings, where ν=1.

Conclusion

A model of the 40m LIGO prototype in a dual-recycling configuration was successfully created using Twiddle. A length sensing scheme, developed by Jim Mason, was implemented in this model and the resulting values from this scheme were simulated and recorded. The signal-recycling capabilities of the configuration was verified. The lengths and optical parameters of the interferometer were varied to optimize the length sensing signals and the signal-recycling properties of the system. These parameters can now be implemented at the 40m LIGO prototype and eventually at the 4km LIGO detectors.

Since the completion of this project, the most likely length sensing scheme for LIGO II has been changed. There will no longer be a subcarrier, but rather, a second set of sidebands with modulation frequency many times that of the first sidebands. There will also be a DC offset put into the arms to allow some carrier to leak out the dark port. The other characteristics of the system are very similar, but it now needs to be modeled with Twiddle and appropriate lengths and optical parameters need to be found before implementation at the 40m.

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